4 Quadratic Equations

Fastrack Revision

- **Quadratic Equation:** An equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, is called a quadratic equation in variable x. *e.g.*, $4x^2 + 2x + 3 = 0$, $5x^2 - 7 = 0$, $x^2 - 7x = 0$, etc.

Knowledge BOOSTER

1. *Each quadratic equation can have atmost two real roots.*
2. *Some of the quadratic equations do not have even a single real root.*

- **Roots or Solution of a Quadratic Equation:** A real number α (alpha) is called a root of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, if $a\alpha^2 + b\alpha + c = 0$.

It means $x = \alpha$ satisfies the equation $ax^2 + bx + c = 0$ or $x = \alpha$ is the root or solution of the quadratic equation $ax^2 + bx + c = 0$.

- **Solving a Quadratic Equation:** There are two methods to solve a quadratic equation:

1. Factorisation Method:

(a) Factorisation of the quadratic equation
$$x^2 + bx + c = 0.$$

(i) First find two numerical factors of the constant term c whose algebraic sum is equal to b, *i.e.*, the coefficient of x. Let two such factors of c be p and q.

i.e., $c = p \times q$ while $b = p + q$

(ii) Write the middle term bx of the given quadratic equation as a sum $(px + qx)$ in the following form:

$$x^2 + bx + c = x^2 + px + qx + pq = 0$$
$$\Rightarrow \quad x(x + p) + q(x + p) = 0$$
$$\Rightarrow \quad (x + p)(x + q) = 0$$
$$\Rightarrow \quad x + p = 0 \text{ or } x + q = 0$$
$$\Rightarrow \quad x = -p \text{ or } x = -q$$

(b) Factorisation of the quadratic equation $ax^2 + bx + c = 0$ where $a \neq 1$ and $a \neq 0$.

(i) Find the product ac of the coefficient of x^2 and constant term.

(ii) Find two numerical factors of this product ac whose algebraic sum is equal to b, *i.e.*, the coefficient of x. Let two such factors of ac be p and q.

i.e., $\quad ac = p \times q$

and $\quad b = p + q$

(iii) Write the middle term bx of the given quadratic equation as a sum $(px + qx)$ in the following form:

$$ax^2 + bx + c = ax^2 + px + qx + c = 0$$
$$\Rightarrow \quad ax^2 + px + qx + \frac{pq}{a} = 0$$
$$\Rightarrow \quad \frac{1}{a}\{a^2x^2 + apx + aqx + pq\} = 0$$
$$\Rightarrow \quad \frac{1}{a}\{ax(ax + p) + q(ax + p)\} = 0$$
$$\Rightarrow \quad \frac{1}{a}(ax + p)(ax + q) = 0$$
$$\Rightarrow \quad ax + p = 0 \text{ or } ax + q = 0$$
$$\Rightarrow \quad x = \frac{-p}{a} \text{ and } x = \frac{-q}{a} \quad (\because a \neq 0)$$

2. Quadratic Formula (Shridharacharya Formula): Let a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has two roots α and β, then

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and
$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

- **Discriminant:** For the quadratic equation $ax^2 + bx + c = 0$, the expression $D = b^2 - 4ac$ is called the discriminant. The roots are determined in terms of discriminant as

$$\alpha = \frac{-b + \sqrt{D}}{2a}$$

and
$$\beta = \frac{-b - \sqrt{D}}{2a}$$

- **Nature of Roots:**

Case I: If $D \geq 0$, *i.e.*, $b^2 - 4ac \geq 0$, then roots are real.

Case II: If $D > 0$, *i.e.*, $b^2 - 4ac > 0$, then roots are real and distinct.

Case III: If $D = 0$, *i.e.*, $b^2 - 4ac = 0$, then roots are real and equal and each root is equal to $\frac{-b}{2a}$.

Case IV: If $D < 0$, *i.e.*, $b^2 - 4ac < 0$, then roots are not real, *i.e.*, imaginary.

Practice Exercise

Multiple Choice Questions

Q 1. Identify the quadratic equation in the following options:

a. $(x-3)^2 + 1 = x^2 + 3$
b. $(x-2)(x-1) = x^2 + 2$
c. $\frac{x^3 + x^2}{x} = 4$
d. $x + \frac{4}{x} = x^2$

Q 2. If the quadratic equation $ax^2 + bx + c = 0$ has two real and equal roots, then 'c' is equal to: *[CBSE 2023]*

a. $-\frac{b}{2a}$
b. $\frac{b}{2a}$
c. $\frac{-b^2}{4a}$
d. $\frac{b^2}{4a}$

Q 3. The roots of the quadratic equation $x^2 - 0.04 = 0$ are: *[CBSE 2020]*

a. ± 0.2
b. ± 0.02
c. 0.4
d. 2

Q 4. The roots of the equation $x^2 + 3x - 10 = 0$ are: *[CBSE 2023]*

a. 2, −5
b. −2, 5
c. 2, 5
d. −2, −5

Q 5. The discriminant of the quadratic equation $2x^2 - 5x - 3 = 0$ is: *[CBSE 2023]*

a. 1
b. 49
c. 7
d. 19

Q 6. If $x = 3$ is one of the roots of the quadratic equation $x^2 - 2kx - 6 = 0$, then the value of k is: *[CBSE SQP 2023-24]*

a. $-\frac{1}{2}$
b. $\frac{1}{2}$
c. 3
d. 2

Q 7. Write the nature of roots of the quadratic equation $9x^2 - 6x - 2 = 0$. *[CBSE SQP 2023-24]*

a. No real roots
b. 2 equal real roots
c. 2 distinct real roots
d. More than 2 real roots

Q 8. If $x = \frac{1}{\sqrt{3}}$ is the root of $kx^2 + (\sqrt{3} - \sqrt{2})x - 1 = 0$, then k is equal to:

a. $\sqrt{6}$
b. $2\sqrt{6}$
c. $-\sqrt{6}$
d. $3\sqrt{6}$

Q 9. The solution of the equation $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$, $x \neq 3, -5$ is/are:

a. 8
b. −9, 7
c. 7, 9
d. −9, −7

Q 10. If a root of the equation $x^2 + ax + 3 = 0$ is 1, then its other root will be:

a. 3
b. − 3
c. 2
d. − 2

Q 11. The roots of the equation $5x + \frac{1}{x} = 6$, $x \neq 0$ are:

a. $\frac{1}{3}, 2$
b. $\frac{1}{4}, 1$
c. $\frac{1}{5}, 1$
d. $-1, -\frac{1}{5}$

Q 12. Which of the following equations has two distinct real roots? *[NCERT EXEMPLAR]*

a. $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$
b. $x^2 + x - 5 = 0$
c. $x^2 - 3x + 2\sqrt{2} = 0$
d. $5x^2 - 3x + 1 = 0$

Q 13. Which of the following equations has no real roots? *[NCERT EXEMPLAR]*

a. $x^2 - 4x + 3\sqrt{2} = 0$
b. $x^2 + 4x - 3\sqrt{2} = 0$
c. $x^2 - 4x - 3\sqrt{2} = 0$
d. $3x^2 + 4\sqrt{3}x + 4 = 0$

Q 14. Value (s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots, is: *[CBSE SQP 2023-24]*

a. 0 only
b. 4
c. 8 only
d. 0, 8

Q 15. If the quadratic equation $9x^2 + bx + \frac{1}{4} = 0$ has equal roots, then the value of b is: *[CBSE 2023]*

a. 0
b. −3 only
c. 3 only
d. ± 3

Q 16. The quadratic equation $x^2 + dx - 8 = 0$ has:

a. no real roots
b. real and distinct roots
c. real and equal roots
d. real and imaginary roots

Q 17. If the equation $(2k + 1)x^2 + 2(k + 3)x + (k + 5) = 0$ has real and equal roots, then the equation forms in terms of k is:

a. $k^2 + 4k - 5 = 0$
b. $k^2 + 5k - 4 = 0$
c. $2k^2 - 5k + 4 = 0$
d. $k^2 - 5k + 4 = 0$

Q 18. If the real roots of the equation $x^2 - bx + 1 = 0$ are not possible, then:

a. $-3 < b < 3$
b. $-2 < b < 2$
c. $b > 2$
d. $b < -2$

Q 19. If a root of the equation $x^2 + bx + 12 = 0$ is 2 and the roots of the equation $x^2 + bx + q = 0$ are equal, then the value of q is:

a. 8
b. −8
c. 16
d. −16

Assertion & Reason Type Questions

Directions (Q. Nos. 20-25): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 20. Assertion (A): $(3x-2)^2 - 9x^2 + 10 = 0$ is a quadratic equation.

Reason (R): $x = 0, 4$ are the roots of the equation $3x^2 - 12x = 0$.

Q 21. Assertion (A): If $5+\sqrt{7}$ is a root of a quadratic equation with rational coefficients, then its other root is $5-\sqrt{7}$.

Reason (R): Surd roots of a quadratic equation with rational coefficients occur in conjugate pairs.

[CBSE 2023]

Q 22. Assertion (A): One of the root of the equation $2x^2 + 5x - 3 = 0$ is $\frac{1}{2}$.

Reason (R): Roots of the quadratic equation $ax^2 + bx + c = 0$ can be determined by using the formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Q 23. Assertion (A): The value of $k = -\frac{1}{4}$, if one root of the quadratic equation $5x^2 - x + 3k = 0$ is $\frac{1}{2}$.

Reason (R): The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has atmost two roots.

Q 24. Assertion (A): The roots of the quadratic equation $x^2 + 5x + 7 = 0$ are imaginary.

Reason (R): In quadratic equation $ax^2 + bx + c = 0$, if $D = b^2 - 4ac < 0$, then roots are said to be imaginary.

Q 25. Assertion (A): The equation $9x^2 + 3kx + 4 = 0$ has equal roots for $k = \pm 4$.

Reason (R): If discriminant 'D' of a quadratic equation equals to zero, then the roots of quadratic equation are real and equal.

Fill in the Blanks Type Questions

Q 26. If the product of two consecutive positive integers is 306 and we need to find the integers, then the situation can be represented in terms of x as .... *[NCERT EXEMPLAR]*

Q 27. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is .... *[NCERT EXEMPLAR]*

Q 28. The roots of the quadratic equation $4x^2 + 2x - 3 = 0$ has the nature as .... *[NCERT EXEMPLAR]*

Q 29. Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots, is /are .... *[NCERT EXEMPLAR]*

Q 30. If discriminant of any quadratic equation is less than zero, then quadratic equation has .... roots.

True/False Type Questions

Q 31. $(x+1)^2 = 2(x-3)$ is a type of quadratic equation. *[NCERT EXERCISE]*

Q 32. The equation $2x^2 - 7x + 6 = 0$ has 2 as a root.

Q 33. The roots of the equation $x^2 - kx - 9 = 0$ has equal and opposite sign, when $k = 0$.

Q 34. If $D > 0$, *i.e.*, $b^2 - 4ac > 0$, then the roots are real and equal.

Q 35. The non-zero value of k for which the roots of the quadratic equation $9x^2 - 3kx + k = 0$ are real and equal, is 3.

Solutions

1. (c) Consider the equation is $\frac{x^3 + x^2}{x} = 4$

$\Rightarrow \quad x^2 + x = 4$

which is a quadratic equation.

2. (d) Given quadratic equation is

$$ax^2 + bx + c = 0$$

It has two real and equal roots then its discriminant should be zero.

$\therefore \quad D = B^2 - 4AC \Rightarrow b^2 - 4ac = 0$

$\Rightarrow \quad 4ac = b^2 \Rightarrow c = \frac{b^2}{4a}$

3. (a) Given, $x^2 - 0.04 = 0$

$\Rightarrow \quad x^2 = 0.04$

$\Rightarrow \quad x^2 = \pm 0.2$

4. (a) Given quadratic equation is

$$x^2 + 3x - 10 = 0$$

$\Rightarrow \quad x^2 + (5-2)x - 10 = 0$

(by splitting the middle term)

$\Rightarrow \quad x^2 + 5x - 2x - 10 = 0$

$\Rightarrow \quad x(x+5) - 2(x+5) = 0$

$\Rightarrow \quad (x+5)(x-2) = 0$

$\Rightarrow \quad x + 5 = 0 \quad \text{or} \quad x - 2 = 0$

$\Rightarrow \quad x = -5, 2$

Hence, the roots of the given quadratic equation are $-5, 2$.

5. (b) Given quadratic equation is

$$2x^2 - 5x - 3 = 0$$

On comparing with standard quadratic equation $ax^2 + bx + c = 0$, we get

$$a = 2, b = -5 \text{ and } c = -3$$

$\therefore$ Discriminant $(D) = b^2 - 4ac = (-5)^2 - 4(2)(-3)$

$= 25 + 24 = 49$

6. (b) Given, $x = 3$ is a root of the equation $x^2 - 2kx - 6 = 0$.

Therefore, the value of x satisfies the given equation.

So, put $x = 3$ in the given equation, we get

$$(3)^2 - 2k(3) - 6 = 0$$

$\Rightarrow \quad 9 - 6k - 6 = 0$

$\Rightarrow \quad 6k = 3 \Rightarrow k = \frac{3}{6} = \frac{1}{2}$

7. (c) Given quadratic equation is $9x^2 - 6x - 2 = 0$.

On comparing with standard quadratic equation $ax^2 + bx + c = 0$, we get

$a = 9, b = -6$ and $c = -2$

Now, discriminant $(D) = b^2 - 4ac$

$= (-6)^2 - 4 \times 9 \times (-2)$

$= 36 + 72 = 108 > 0$

Hence, given quadratic has two distinct real roots.

8. (a) Given, $x = \frac{1}{\sqrt{3}}$ is a root of the equation $kx^2 + (\sqrt{3} - \sqrt{2})x - 1 = 0$. Therefore, the value of x satisfies the given equation.

So, put $x = \frac{1}{\sqrt{3}}$ in the given equation, we get

$$k\left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3} - \sqrt{2})\left(\frac{1}{\sqrt{3}}\right) - 1 = 0$$

$$\Rightarrow \quad k\left(\frac{1}{3}\right) + \left(1 - \sqrt{\frac{2}{3}}\right) - 1 = 0$$

$$\Rightarrow \quad k\left(\frac{1}{3}\right) - \sqrt{\frac{2}{3}} = 0$$

$$\Rightarrow \quad k = \sqrt{\frac{2}{3}} \times 3 = \sqrt{2} \times \sqrt{3} = \sqrt{6}$$

9. (b) Given equation is

$$\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{(x+5)-(x-3)}{(x+5)(x-3)} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{8}{(x+5)(x-3)} = \frac{1}{6}$$

$\Rightarrow \quad (x+5)(x-3) = 48$

$\Rightarrow \quad x^2 + 2x - 15 = 48$

$\Rightarrow \quad x^2 + 2x - 63 = 0$

$\Rightarrow \quad x^2 + (9-7)x - 63 = 0$

$\Rightarrow \quad x^2 + 9x - 7x - 63 = 0$

$\Rightarrow \quad x(x+9) - 7(x+9) = 0$

$\Rightarrow \quad (x-7)(x+9) = 0 \Rightarrow x = 7, -9$

10. (a) Given, 1 is a root of the equation $x^2 + ax + 3 = 0$.

$\therefore \quad (1)^2 + a(1) + 3 = 0 \quad \Rightarrow \quad a = -4$

$\therefore$ The given equation becomes $x^2 - 4x + 3 = 0$.

By using quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+4 \pm \sqrt{(-4)^2 - 4 \times 3}}{2 \times 1} = \frac{4 \pm \sqrt{16 - 12}}{2}$$

$$= \frac{4 \pm \sqrt{4}}{2} = \frac{4 \pm 2}{2} = \frac{6}{2}, \frac{2}{2} = 3, 1$$

Hence, other root is 3.

11. (c) Given equation is

$$5x + \frac{1}{x} = 6, \ x \neq 0$$

$\Rightarrow \quad 5x^2 + 1 = 6x$

$\Rightarrow \quad 5x^2 - 6x + 1 = 0$

$\Rightarrow \quad 5x^2 - (5+1)x + 1 = 0$

$\Rightarrow \quad 5x^2 - 5x - x + 1 = 0$

$\Rightarrow \quad 5x(x-1) - 1(x-1) = 0$

$\Rightarrow \quad (5x-1)(x-1) = 0$

$\Rightarrow \quad x = \frac{1}{5}, 1$

12. (b)

TR!CK

The condition for quadratic equation has two distinct real roots, if

$D > 0$ *i.e.,* $b^2 - 4ac > 0$

Consider equation is $x^2 + x - 5 = 0$.

Now, discriminant $(D) = b^2 - 4ac$

$= (1)^2 - 4 \times 1 \times (-5)$

$= 1 + 20 = 21 > 0$

Hence, this quadratic equation has two distinct real roots.

13. (a)

TR!CK

The condition for quadratic equation has no real roots, if

$D < 0$ *i.e.,* $b^2 - 4ac < 0$.

Consider equation $x^2 - 4x + 3\sqrt{2} = 0$.

Now, discriminant, $D = b^2 - 4ac$

$= (-4)^2 - 4 \times 1 \times (3\sqrt{2})$

$= 16 - 12\sqrt{2} = -0.968 < 0$

Hence, no real roots exist.

14. (d) Given quadratic equation is $2x^2 - kx + k = 0$.

As the given quadratic equation has equal roots. Therefore, its discriminant equals to zero.

i.e., $\quad D = 0$

or $\quad b^2 - 4ac = 0$

On comparing given equation with $ax^2 + bx + c = 0$, we get

$a = 2, b = -k$ and $c = k$

$\therefore \quad (-k)^2 - 4 \times 2 \times k = 0 \quad \Rightarrow \quad k^2 - 8k = 0$

$\Rightarrow \quad k(k-8) = 0 \quad \Rightarrow \quad k = 0, 8$

15. (d)

TR!CK

The condition for equation $ax^2 + bx + c = 0$ *has equal roots, is* $D = b^2 - 4ac = 0$.

Given equation is $9x^2 + bx + \frac{1}{4} = 0$

Here, $A = 9, B = b$ and $C = \frac{1}{4}$

Since, given equation has equal roots.

Therefore,

Discriminant $(D) = 0 \Rightarrow B^2 - 4AC = 0$

$\Rightarrow b^2 - 4 \times 9 \times \frac{1}{4} = 0$

$\Rightarrow b^2 - 9 = 0 \Rightarrow b^2 - (3)^2 = 0$

$\Rightarrow b = \pm 3$

16. (b) Given quadratic equation is $x^2 + dx - 8 = 0$.

Here, $a = 1, b = d$ and $c = -8$

Now, discriminant, $D = b^2 - 4ac$

$= (d)^2 - 4 \times 1 \times (-8)$

$= d^2 + 32 > 0 \quad (\because d^2 > 0)$

Hence, given equation has real and distinct roots.

17. (b) Given quadratic equation is

$(2k + 1)x^2 + 2(k + 3)x + (k + 5) = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 2k + 1, b = 2(k + 3)$ and $c = k + 5$

Since, the given equation has real and distinct roots.

Therefore,

$$D = 0$$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow [2(k + 3)]^2 - 4 \times (2k + 1) \times (k + 5) = 0$

$\Rightarrow 4(k + 3)^2 - 4(2k^2 + 11k + 5) = 0$

$\Rightarrow 4(k^2 + 9 + 6k) - 4(2k^2 + 11k + 5) = 0$

$\Rightarrow k^2 + 9 + 6k - (2k^2 + 11k + 5) = 0$

$\Rightarrow -k^2 - 5k + 4 = 0$

$\Rightarrow k^2 + 5k - 4 = 0$

18. (b) Given quadratic equation is $x^2 - bx + 1 = 0$.

Here, $A = 1, B = -b$ and $C = 1$

Since, roots of given equation are not real.

$\therefore D < 0$

$\Rightarrow B^2 - 4AC < 0$

$\therefore (-b)^2 - 4 \times 1 \times 1 < 0$

$\Rightarrow (b)^2 < 4$

$\Rightarrow (b)^2 < (2)^2$

$\Rightarrow -2 < b < 2$

19. (c) Given, 2 is a root of the equation $x^2 + bx + 12 = 0$.

Therefore, put $x = 2$ in this equation, we get

$(2)^2 + b(2) + 12 = 0$

$\Rightarrow 4 + 2b + 12 = 0$

$\Rightarrow 2b = -16$

$\Rightarrow b = -8$

Put $b = -8$ in the quadratic equation $x^2 + bx + q = 0$, we get

$x^2 - 8x + q = 0 \quad ...(1)$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = -8$ and $c = q$

Also eq. (1) has equal roots.

$\therefore$ Discriminant $(D) = b^2 - 4ac = 0$

$\Rightarrow (-8)^2 - 4 \times 1 \times q = 0$

$\Rightarrow 64 - 4q = 0$

$\Rightarrow q = \frac{64}{4} = 16$

20. (d) **Assertion (A):** Given equation is

$(3x - 2)^2 - 9x^2 + 10 = 0$

$\Rightarrow 9x^2 + 4 - 12x - 9x^2 + 10 = 0$

$\Rightarrow 14 - 12x = 0$, which is not a quadratic equation.

Thus, Assertion (A) is false.

Reason (R): Given quadratic equation is $3x^2 - 12x = 0$.

$\Rightarrow 3x(x - 4) = 0$

$\Rightarrow x = 0, 4$

Thus, Reason (R) is true.

Hence, Assertion (A) is false but Reason (R) is true.

21. (a) The quadratic formula, which was derived by completing the square, tells us:

$$x_1 = \frac{-b}{2a} + \frac{\sqrt{D}}{2a} \text{ and } x_2 = \frac{-b}{2a} - \frac{\sqrt{D}}{2a}$$

where $(D) = b^2 - 4ac$

Here, x_1 and x_2 are real conjugates of one another if (D) is positive but not a perfect square.

Given, one of the roots of quadratic equation having rational coefficients is $5 + \sqrt{7}$.

So, the second root will be $5 - \sqrt{7}$.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

22. (a) **Assertion (A):** Given equation is $2x^2 + 5x - 3 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$a = 2, b = 5$ and $c = -3$

By using quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{(5)^2 - 4 \times 2 \times (-3)}}{2 \times 2}$$

$$= \frac{-5 \pm \sqrt{25 + 24}}{4} = \frac{-5 \pm \sqrt{49}}{4}$$

$$= \frac{-5 \pm 7}{4} = \frac{-5 + 7}{4} \text{ or } \frac{-5 - 7}{4}$$

$$= \frac{2}{4} \text{ or } -\frac{12}{4} = \frac{1}{2} \text{ or } -3$$

Thus, one of the root of the given equation is $\frac{1}{2}$.

So, Assertion (A) is true.

Reason (R): It is also true that roots are determined by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

23. (b) **Assertion (A):** Given quadratic equation is

$5x^2 - x + 3k = 0$

Since, $\frac{1}{2}$ is a root of the equation $5x^2 - x + 3k = 0$.

Therefore, put $x = \frac{1}{2}$, we get

$$5\left(\frac{1}{2}\right)^2 - \frac{1}{2} + 3k = 0$$

$$\Rightarrow \quad \frac{5}{4} - \frac{1}{2} + 3k = 0$$

$$\Rightarrow \quad 3k = \frac{1}{2} - \frac{5}{4} \quad \Rightarrow 3k = \frac{2-5}{4}$$

$$\Rightarrow \quad k = \frac{-3}{3 \times 4} \quad \Rightarrow \quad k = -\frac{1}{4}$$

Thus, Assertion (A) is true.

Reason (R): It is also true to say that any quadratic equation has atmost two roots.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

24. (a) **Assertion (A):** Given quadratic equation is

$$x^2 + 5x + 7 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 5 \text{ and } c = 7$$

Now, discriminant $D = b^2 - 4ac$

$$= (5)^2 - 4 \times 1 \times 7$$

$$= 25 - 28 = -3 < 0$$

So, roots of given equation are imaginary.

Thus, Assertion (A) is true.

Reason (R): It is also true that, the condition of imaginary root is $D < 0$.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

25. (a) **Assertion (A):** Given equation is

$$9x^2 + 3kx + 4 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 9, b = 3k \text{ and } c = 4.$$

The condition for equal roots is $D = 0$.

$$\therefore \quad b^2 - 4ac = 0$$

$$\Rightarrow \quad (3k)^2 - 4 \times 9 \times 4 = 0$$

$$\Rightarrow \quad 9k^2 - 9 \times 16 = 0$$

$$\Rightarrow \quad k^2 = 16 \quad \Rightarrow \quad k = \pm 4$$

Thus, Assertion (A) is true.

Reason (R): It is also true and it is the correct explanation of Assertion (A).

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

26. $x(x+1) = 306$ or $x^2 + x - 306 = 0$

27. Given, $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$.

$$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$

$$\Rightarrow \quad \frac{k}{2} = \frac{5}{4} - \frac{1}{4}$$

$$\Rightarrow \quad \frac{k}{2} = \frac{4}{4} \quad \Rightarrow \quad k = 2$$

28. Given quadratic equation is $4x^2 + 2x - 3 = 0$.

Here $a = 4$, $b = 2$ and $c = -3$

Now, discriminant, $D = b^2 - 4ac$

$$= (2)^2 - 4 \times 4 \times (-3)$$

$$= 4 + 48 = 52 > 0$$

Hence, roots are real and distinct.

29. Given quadratic equation is $2x^2 - kx + k = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, \ b = -k \text{ and } c = k.$$

Since, equation has equal roots. Therefore,

Discriminant $(D) = b^2 - 4ac = 0$

$$\Rightarrow \quad (-k)^2 - 4 \times 2\,(k) = 0$$

$$\Rightarrow \quad k(k-8) = 0$$

$$\Rightarrow \quad k = 0, 8$$

30. imaginary

31. Given, $(x+1)^2 = 2(x-3)$

$$\Rightarrow \quad x^2 + 2x + 1 = 2x - 6$$

$\Rightarrow$ $x^2 + 7 = 0$, which is a quadratic equation.

Hence, given statement is true.

32. Given, $2x^2 - 7x + 6 = 0$

Put $x = 2$

$$\therefore \quad 2(2)^2 - 7(2) + 6 = 8 - 14 + 6$$

$= 0$, which is true.

Hence, given statement is true.

33. Suppose $k = 0$. Therefore, $x^2 - 9 = 0 \Rightarrow x = \pm 3$

So, the roots are equal and opposite sign.

Hence, given statement is true.

34. If $D > 0$, then roots of the quadratic equation are real and distinct.

Hence, given statement is false.

35. Given quadratic equation is $9x^2 - 3kx + k = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 9, b = -3k \text{ and } c = k$$

Since, the roots of given quadratic equation has real and equal.

$$\therefore D = 0 \quad \Rightarrow \quad b^2 - 4ac = 0$$

$$\Rightarrow \quad (-3k)^2 - 4 \times 9 \times k = 0$$

$$\Rightarrow \quad 9k^2 - 36k = 0$$

$$\Rightarrow \quad 9k(k-4) = 0$$

$$\Rightarrow \quad k = 0, 4$$

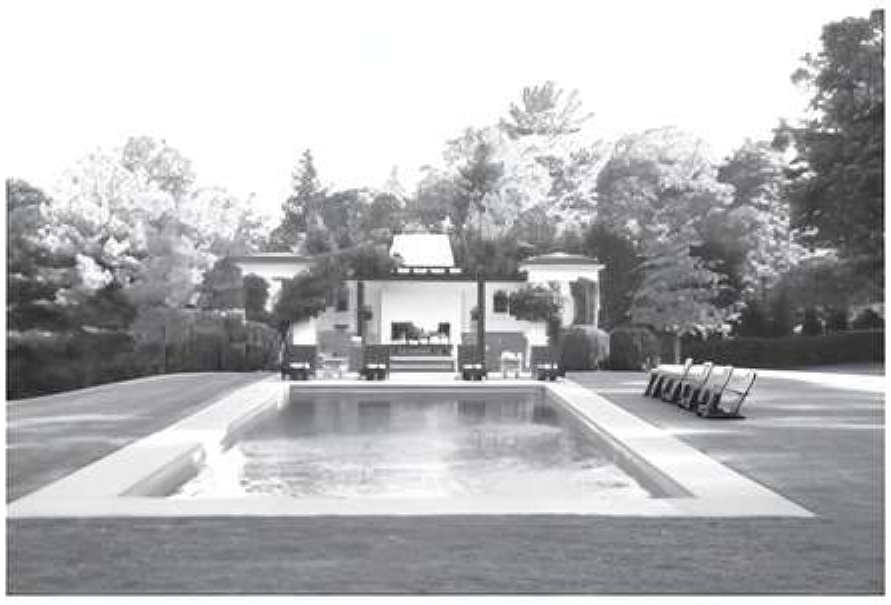
Hence, given statement is false.

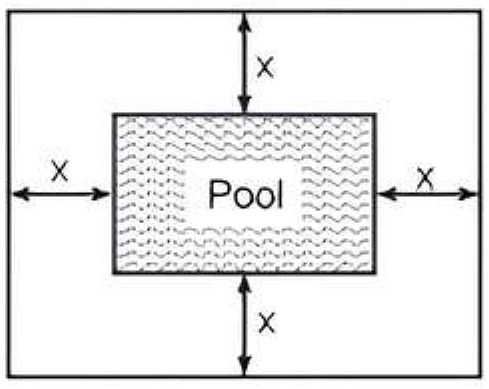
Case Study Based Questions

Case Study 1

Noida authority decided to make a park for the people, so that the persons make them physically fit and take a fresh air.

A grassy park is in the form of rectangle having length 20 m and breadth 14 m. At the centre of the park, there is a rectangular pool, which is at a distance of equal width around it, there is a path having an area of 120 m^2.





Based on the given information, solve the following questions:

Q 1. If the centre pool is at x metre distance from around the park, then length and breadth of the pool (in metre) will be:

a. $(20-4x), (14-4x)$ b. $(20-x), (14-x)$
c. $(20-2x), (14-2x)$ d. $(20+2x), (14+2x)$

Q 2. If the area of path is 120 m^2, then the quadratic equation in terms of x is:

a. $x^2-17x+48=0$ b. $x^2-17x+30=0$
c. $x^2-17x+36=0$ d. $x^2-16x+138=0$

Q 3. Find the nature of the roots of the equation formed in part 2.

a. Real and equal b. Real and distinct
c. Imaginary d. None of these

Q 4. Width of the pool is:

a. 6 m b. 2 m c. 12 m d. 7 m

Q 5. The area of the rectangular pool is:

a. 30 m^2 b. 40 m^2 c. 46 m^2 d. 160 m^2

Solutions

1. Given, length and breadth of a park are $l_1 = 20$ m and $b_1 = 14$ m.
   Then the length and breadth of the pool will be $(20-2x)$ m and $(14-2x)$ m.
   So, option (c) is correct.

2. Given, area of path = 120 m^2
   $\therefore$ Area of rectangular park
   – Area of rectangular pool = 120
   $$\Rightarrow 20 \times 14 - (20-2x)(14-2x) = 120$$
   $$\Rightarrow 280 - (280 - 40x - 28x + 4x^2) = 120$$
   $$\Rightarrow 68x - 4x^2 = 120$$
   $$\Rightarrow 4x^2 - 68x + 120 = 0$$
   $$\Rightarrow x^2 - 17x + 30 = 0$$
   So, option (b) is correct.

3. Since, quadratic equation is $x^2-17x+30=0$.
   On comparing with $ax^2+bx+c=0$, we get
   $a = 1, b = -17$ and $c = 30$
   $$\therefore \text{Discriminant } (D) = b^2 - 4ac$$
   $$= (-17)^2 - 4 \times 1 \times (30)$$
   $$= 289 - 120 = 169 > 0$$
   Here, discriminant is positive, so roots are real and distinct.
   So, option (b) is correct.

4. Quadratic equation is $x^2-17x+30=0$.
   Here, $a = 1, b = -17, c = 30$
   Using quadratic formula,
   $$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{+17 \pm \sqrt{169}}{2 \times 1} = \frac{17 \pm 13}{2}$$
   $$\Rightarrow \quad x = \frac{17+13}{2} \text{ or } x = \frac{17-13}{2}$$
   $$\Rightarrow \quad x = \frac{30}{2} \text{ or } x = \frac{4}{2}$$
   $$\Rightarrow \quad x = 15 \text{ or } x = 2$$
   If we consider $x = 15$, then $l_2 = 20 - 2 \times 15$
   $= 20 - 30 = -10$.
   which is not possible.
   $\therefore$ We consider only $x = 2$ m.
   Thus, width of the pool is 2 m.
   So, option (b) is correct.

5. The area of the rectangular pool $= l_2 \times b_2$
   $$= (20-2x)(14-2x)$$
   $$= (20 - 2 \times 2)(14 - 2 \times 2) \quad \text{(put } x = 2\text{)}$$
   $$= 16 \times 10 = 160 \text{ m}^2$$
   So, option (d) is correct.

Case Study 2

In cricket match of world cup 2016, Ashwin took 2 wickets less than twice the number of wickets taken by Ishant. The product of the numbers of wickets taken by these two is 24.

Based on the above information, solve the following questions:

Q 1. If Ishant took x wickets in the world cup, then wickets taken by Ashwin are:

a. $(2x+2)$ b. $(2x-2)$
c. $(2x-1)$ d. $(2x-5)$

Q 2. The given statement represents in equation form as:

a. $x^2-3x+10=0$ b. $x^2+x+12=0$
c. $x^2-x-12=0$ d. $x^2-2x+12=0$

Q 3. If quadratic equation has real and equal roots, then condition of discriminant D is:

a. $D < 0$ b. $D = 0$

c. $D \geq 0$ d. $D > 0$

Q 4. The nature of roots of the equation formed by given statement is:

a. real and equal b. real and distinct

c. imaginary d. real and imaginary

Q 5. The number of wickets taken by Ashwin is:

a. 3 b. 4 c. 6 d. 2

Solutions

1. As, Ishant took x wickets in the world cup, then Ashwin took 2 wickets less than twice the number of wickets taken by Ishant *i.e.,* $(2x - 2)$ wickets.

So, option (b) is correct.

2. According to the given statement, the product of the number of wickets taken by these two players = 24

$\therefore \quad x(2x - 2) = 24$

$\Rightarrow \quad 2x^2 - 2x = 24 \quad \Rightarrow \quad x^2 - x - 12 = 0$

So, option (c) is correct.

3. The condition for discriminant having real and equal roots is $D = 0$.

So, option (b) is correct.

4. From part 2, quadratic equation is $x^2 - x - 12 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = -1$ and $c = -12$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$= (-1)^2 - 4 \times 1 \times (-12)$

$= 1 + 48 = 49 > 0$

Here, discriminant is positive, so roots are real and distinct.

So, option (b) is correct.

5. As quadratic equation is

$x^2 - x - 12 = 0$

$\Rightarrow \quad x^2 - (4 - 3)x - 12 = 0$

$\Rightarrow \quad x^2 - 4x + 3x - 12 = 0$

$\Rightarrow \quad x(x - 4) + 3(x - 4) = 0$

$\Rightarrow \quad (x - 4)(x + 3) = 0$

$\Rightarrow \quad x - 4 = 0$ or $x + 3 = 0$

$\Rightarrow \quad x = 4$ or $x = -3$

$\therefore \quad x = 4$ ($\because x = -3$, rejected because quantity of wicket cannot be negative)

Thus, Ashwin took the wickets $= 2x - 2$

$= 2(4) - 2 = 8 - 2 = 6$

So, option (c) is correct.

Case Study 3

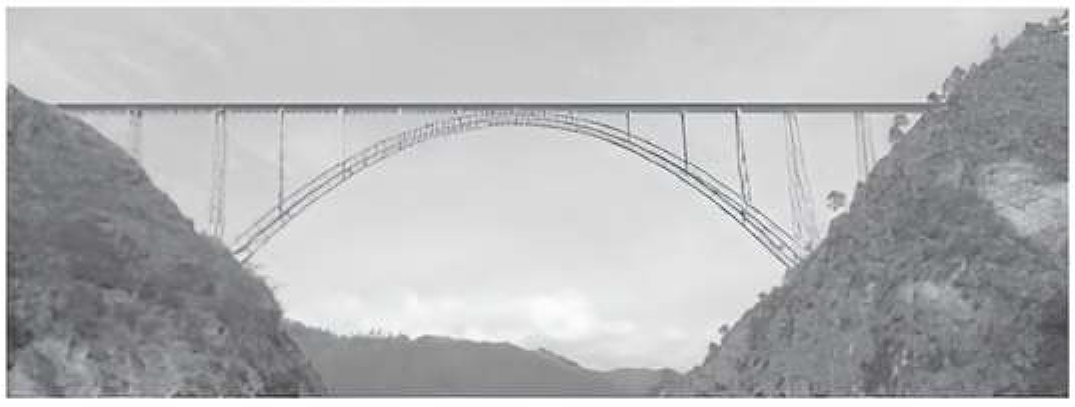
Chenab railway bridge is the World's tallest railway bridge in Jammu and Kashmir Territory, which is constructed on Chenab river. Its shape is a parabolic arch, whose equation is in the form of $ax^2 + bx + c = 0$.

The nature of roots can be defined as:

(i) $D = b^2 - 4ac > 0$, roots are real and distinct.

(ii) $D = b^2 - 4ac = 0$, roots are real and equal.

(iii) $D = b^2 - 4ac < 0$, roots are imaginary.



Based on the above information, solve the following questions:

Q 1. Find the nature of roots of the equation $5x^2 - 4x - 3 = 0$.

Q 2. Identify the type of the roots of quadratic equation $x^2 + 3x + 3 = 0$.

Q 3. Find the value of k in which the equation $3x^2 - 2x + 4k = 0$ has equal roots.

OR

Find the value of k for which the equation $x^2 + 5kx + 16 = 0$ has no real root.

Solutions

1. Given equation is $5x^2 - 4x - 3 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$a = 5, b = -4$ and $c = -3$

Now, discriminant $D = b^2 - 4ac$

$= (-4)^2 - 4 \times 5 \times (-3)$

$= 16 + 60 = 76 > 0$

Hence, roots of given equation are real and distinct.

2. Given quadratic equation is $x^2 + 3x + 3 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = 3$ and $c = 3$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$= (3)^2 - 4 \times 1 \times 3$

$= 9 - 12 = -3 < 0$

Hence, roots of given equation are imaginary.

3. Given quadratic equation is $3x^2 - 2x + 4k = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$a = 3, b = -2$ and $c = 4k$

As the roots of the given equation are equal.

Therefore, discriminant $D = b^2 - 4ac = 0$

$\Rightarrow \quad (-2)^2 - 4 \times 3 \times 4k = 0$

$\Rightarrow \quad 4 - 4 \times 12k = 0$

$\Rightarrow \quad 1 - 12k = 0$

$\Rightarrow \quad k = \frac{1}{12}$

OR

Given equation: $x^2 + 5kx + 16 = 0$

Comparing this equation with general quadratic equation $ax^2 + bx + c = 0$

$a = 1, b = 5k$ and $c = 16$

$\therefore$ Discriminant $(D) = b^2 - 4ac = (5k^2) - 4(1)(16)$

$= 25k^2 - 64$

If the given equation has no real root, then

$$D < 0$$

$$\Rightarrow \quad 25k^2 - 64 < 0 \Rightarrow k^2 < \frac{64}{25}$$

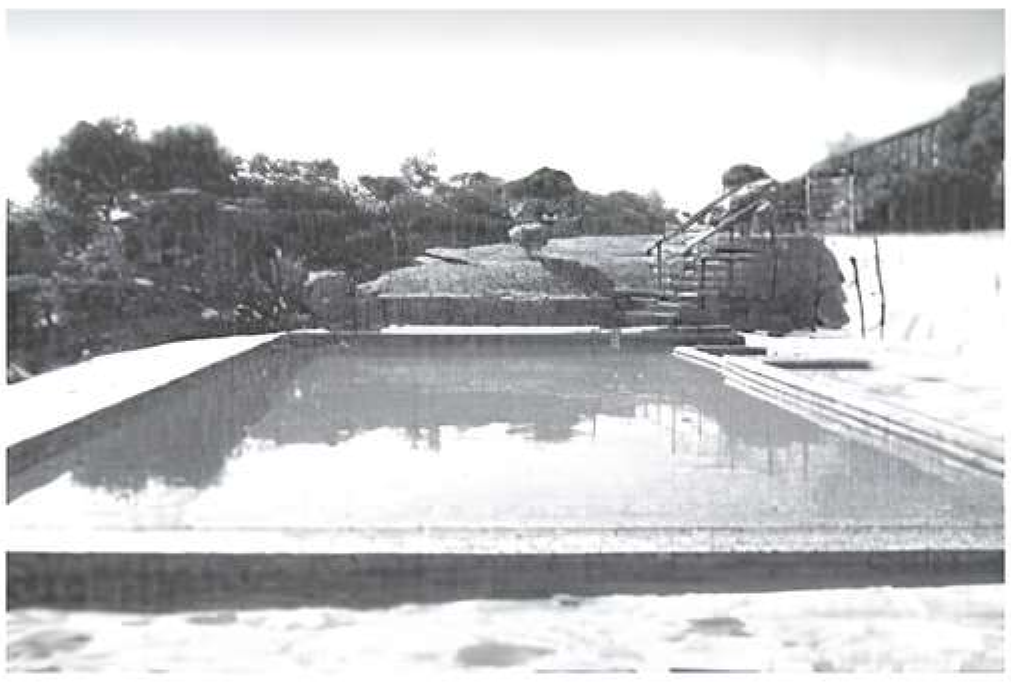
$$\Rightarrow \quad k^2 - \left(\frac{8}{2}\right)^2 < 0$$

$$\Rightarrow \quad -\frac{8}{5} < k < \frac{8}{5} \qquad [\because x^2 - a^2 < 0 \Rightarrow -a < x < a]$$

Therefore, the roots of the given equation will not be real if $-\frac{8}{5} < k < \frac{8}{5}$.

Case Study 4

In the picture given below, one can see a rectangular in-ground swimming pool installed by a family in their backyard. There is a concrete sidewalk around the pool of width x m. The outside edges of the sidewalk measure 7 m and 12 m. The area of the pool is 36 sq. m. *[CBSE 2022 Term -II]*



Based on the given information, solve the following questions:

Q 1. Write the representation of the length and breadth of the pool algebraically.

Q 2. Form a quadratic equation in terms of x.

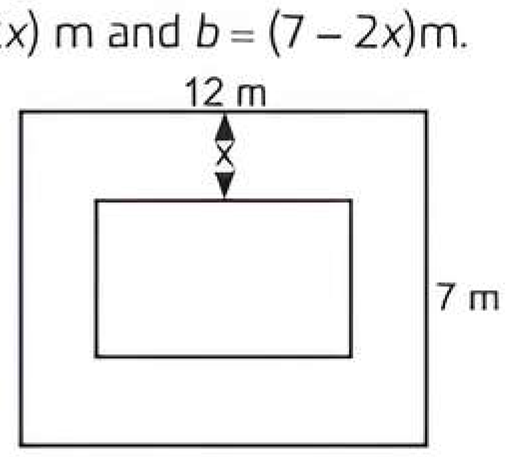
Q 3. Find the length of the sidewalk around the pool.

OR

Find the width of the sidewalk around the pool.

Solutions

1. Given, width of the path be x metres.
Inside, length and breadth of a rectangular pool are $l = (12 - 2x)$ m and $b = (7 - 2x)$m.



2. $\therefore$ Area of the pool $= l \times b$

$$= (12 - 2x) \times (7 - 2x)$$
$$= 84 - 24x - 14x + 4x^2$$
$$= 4x^2 - 38x + 84$$

$\therefore$ The required quadratic equation in terms of x is

$$4x^2 - 38x + 84 = 0$$

or $$2x^2 - 19x + 42 = 0$$

3. We have, area of the pool is 36 m^2.

$$\therefore \quad 4x^2 - 38x + 84 = 36$$
$$\Rightarrow \quad 2x^2 - 19x + 42 = 18$$
$$\Rightarrow \quad 2x^2 - 19x + 24 = 0$$
$$\Rightarrow \quad 2x^2 - (16 + 3)x + 24 = 0$$
$$\Rightarrow \quad 2x^2 - 16x - 3x + 24 = 0$$
$$\Rightarrow \quad 2x(x - 8) - 3(x - 8) = 0$$
$$\Rightarrow \quad (2x - 3)(x - 8) = 0$$
$$\Rightarrow \quad x = \frac{3}{2}, \; x = 8$$

When $x = 8$,

$$b = 7 - 2 \times 8$$
$$= -9 \text{ which is not possible}$$

So, we neglect $x = 8$.

Consider, $x = \frac{3}{2}$.

$\therefore$ Length $\quad l = 12 - 2 \times \frac{3}{2} = 9$ m

So, length of the sidewalk around the pool is 9m.

Or

$\therefore$ Width $\quad b = 7 - 2 \times \frac{3}{2} = 4$ m.

So, width of the sidewalk around the pool is $\frac{3}{2}$m.

Very Short Answer Type Questions

Q 1. Find the sum and product of the roots of the quadratic equation $2x^2 - 9x + 4 = 0$. *[CBSE 2023]*

Q 2. Find the roots of the equation $x^2 - 3x - m(m + 3) = 0$, where m is a constant.

Q 3. If 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then find the value of ab.

Q 4. Find the discriminant of the quadratic equation $4x^2 - 5 = 0$ and hence comment on the nature of roots of the equation. *[CBSE 2023]*

Q 5. What is the nature of roots of quadratic equation $x^2 - 2x + 4 = 0$?

Q 6. Find the value of k for which the quadratic equation $3x^2 + kx + 3 = 0$ has real and equal roots. *[CBSE 2019]*

Q 7. Find the value of k for which the quadratic equation $px(x - 2) + 6 = 0$ has two equal real roots. *[NCERT EXERCISE; CBSE 2023, 19]*

Q 8. For what values of k does the quadratic equation $4x^2 - 12x - k = 0$ have no real roots? *[CBSE 2019]*

Short Answer Type-I Questions

Q 1. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b. *[CBSE 2016]*

Q 2. Solve for x : $9x^2 - 6px + (p^2 - q^2) = 0$. *[CBSE SQP 2022 Term-II]*

Q 3. Solve the following quadratic equation for x:

$$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$$ [CBSE 2022 Term -II]

Q 4. Solve the quadratic equation $x^2 + 2\sqrt{2}x - 6 = 0$ for x. [CBSE 2022 Term-II]

Q 5. Find the sum of reciprocal of the roots of equation $t^2 + 3t - 10 = 0$.

Q 6. Solve the quadratic equation $x^2 - 14x + 24 = 0$. Also find the sum of square of their roots.

Q 7. Find the value of m so that the quadratic equation $mx(5x - 6) = 0$ has two equal roots. [CBSE SQP 2022 Term-II]

Q 8. Find the value of m for which the quadratic equation $(m-1)x^2 + 2(m-1)x + 1 = 0$ has two real and equal roots. [CBSE 2022 Term-II]

Q 9. If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, prove that $2b = a + c$. [CBSE 2015]

Q 10. If Ritu were younger by 5 years than what she really is, then the square of her age would have been 11 more than five times her present age. What is her present age? [CBSE SQP 2022 Term-II]

Q 11. The product of Rehan's age (in years) 5 yr ago and his age 7 yr from now, is one more than twice his present age. Find his present age. [CBSE 2022 Term-II]

Q 12. If the discriminant of the equation $6x^2 - bx + 2 = 0$ is 1, then find the positive value of b. Also, find the roots of the given equation.

Short Answer Type-II Questions

Q 1. If α and β are roots of the quadratic equation $x^2 - 7x + 10 = 0$, find the quadratic equation whose roots are α^2 and β^2. [CBSE 2023]

Q 2. Solve for x: $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$, $x \neq -4, 7$. [NCERT EXERCISE; CBSE 2020, 19]

Q 3. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, then show that $c^2 = a^2(1 + m^2)$. [CBSE 2017]

Q 4. If 3 is a root of the quadratic equation $x^2 - x + k = 0$, find the value of p so that the roots of the equation $x^2 + k(2x + k + 2) + p = 0$ are equal. [CBSE 2016, 15]

Q 5. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

Q 6. The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages (in years) was 124. Determine their present age. [CBSE 2023]

Q 7. The sum of reciprocals of Roohi's age (in years) 3 years ago and 5 years hence from now is $\frac{1}{3}$. Find her present age. [CBSE 2023]

Q 8. The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}$, find the two numbers. [CBSE 2023]

Q 9. The difference of two natural numbers is 3 and the difference of their reciprocals is $\frac{3}{28}$. Find the numbers.

Q 10. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed. [CBSE 2019, 18]

Long Answer Type Questions

Q 1. Solve for x: $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$, $x \neq -1, -2, -4$. [CBSE 2019, 18]

Q 2. The diagonal of a rectangular field is 60 m more than the shorter side. If the longer side is 30 m more than the shorter side, find the length of the sides of the field. [CBSE 2023]

Q 3. The difference of the squares of two numbers is 180. The square of the smaller number is 8 times the greater number. Find the two numbers. [CBSE 2022 Term-II]

Q 4. Sum of the areas of two squares is 157 m^2. If the sum of their perimeters is 68 m, find the sides of the two squares. [CBSE 2019]

Q 5. There are three consecutive positive integers such that the square of the first increased by the product of the other two given 154. What are the integers?

Q 6. A motorboat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. [NCERT EXERCISE; CBSE SQP 2023-24; CBSE 2019, 18, 17, 16]

Q 7. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train. [CBSE 2023; CBSE SQP 2023-24]

Q 8. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. [CBSE SQP 2023-24]

Solutions

Very Short Answer Type Questions

1. Given quadratic equation is

$$2x^2 - 9x + 4 = 0$$

$\therefore$ Sum of the roots

$$= (-1) \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= (-1) \cdot \frac{(-9)}{2} = \frac{9}{2}$$

and product of the roots

$$= (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= (-1)^2 \cdot \frac{4}{2} = 2$$

2. Given quadratic equation is

$$x^2 - 3x - m(m+3) = 0$$

TR!CK

Middle term 3 can be rewritten as $(m + 3) - m$.

$\Rightarrow x^2 - \{(m+3) - m\}x - m(m+3) = 0$

$\Rightarrow x^2 - (m+3)x + mx - m(m+3) = 0$

$\Rightarrow x\{x - (m+3)\} + m\{x - (m+3)\} = 0$

$\Rightarrow \{x - (m+3)\}\{x + m\} = 0$

$\Rightarrow x - (m+3) = 0$ or $x + m = 0$

$\Rightarrow x = (m+3)$ or $x = -m$

$\Rightarrow x = -m, (m+3)$

Hence, the roots are $-m$ and $(m+3)$.

3.

TiP

A real number α is said to be a root of an equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

Since, 1 is a root of both the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then

$a(1)^2 + a(1) + 3 = 0$ and $(1)^2 + (1) + b = 0$

$\Rightarrow a + a + 3 = 0$ and $1 + 1 + b = 0$

$\Rightarrow 2a + 3 = 0$ and $b + 2 = 0$

$\Rightarrow a = \frac{-3}{2}$ and $b = -2$

$\therefore ab = \left(\frac{-3}{2}\right)(-2) = 3$

4. Given quadratic equation is $4x^2 - 5 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$a = 4$, $b = 0$ and $c = -5$

$\therefore$ Discriminant $(D) = b^2 - 4ac$

$= (0)^2 - 4(4)(-5)$

$= 80 > 0$

Since, $D > 0$, so roots are real and distinct.

5. Given quadratic equation is $x^2 - 2x + 4 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = -2$ and $c = 4$

$\therefore$ Discriminant $(D) = b^2 - 4ac$

$= (-2)^2 - 4 \times 1 \times 4$

$= 4 - 16 = -12 < 0$

Since, $D < 0$, so roots are not real, *i.e.*, imaginary.

6. Given quadratic equation is

$$3x^2 + kx + 3 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 3$, $b = k$ and $c = 3$

As given quadratic equation has real and equal roots.

$\therefore D = 0$

$\Rightarrow b^2 - 4ac = 0 \Rightarrow k^2 - 4 \times 3 \times 3 = 0$

$\Rightarrow k^2 - 36 = 0 \Rightarrow k^2 = 36$

$\Rightarrow k = \pm 6$

7. Given equation is $px(x - 2) + 6 = 0$.

$\Rightarrow px^2 - 2px + 6 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = p$, $b = -2p$, $c = 6$

$\therefore D = b^2 - 4ac$

$\Rightarrow D = (-2p)^2 - 4(p)(6)$

$= 4p^2 - 24p$

For equal roots, $D = b^2 - 4ac = 0$

$\therefore 4p^2 - 24p = 0$

$\Rightarrow 4p(p - 6) = 0$

$\Rightarrow p = 0$ or $p = 6$

But $p \neq 0$ ($\because$ in a quadratic equation, $a \neq 0$)

$\therefore p = 6$

COMMON ERR(!)R

Sometimes students write the answer of this question as $p = 0, 6$ but it is wrong. Students should cross check the answer i.e., put the values of p in given quadratic equation.

Here, at $p = 0$, the existence of quadratic equation will vanish. So, the correct answer is $p = 6$.

8. Given quadratic equation is $4x^2 - 12x - k = 0$.

Comparing this equation with general quadratic equation $ax^2 + bx + c = 0$, we get

$a = 4$, $b = -12$, $c = -k$

Now, discriminant $(D) = b^2 - 4ac$

$= (-12)^2 - 4(4)(-k)$

$= 144 + 16k$

If the given equation has no real roots, then

$D < 0$

$\therefore 144 + 16k < 0$

$\Rightarrow 16k < -144$

$\Rightarrow k < -9$

Short Answer Type-I Questions

1. Given, $x = \frac{2}{3}$ and $x = -3$ are the roots of the quadratic equation $ax^2 + 7x + b = 0$.

Therefore, put $x=\frac{2}{3}$ and $x=-3$, we get

$$a\left(\frac{2}{3}\right)^2+7\left(\frac{2}{3}\right)+b=0$$

$$\Rightarrow \quad a\left(\frac{4}{9}\right)+\frac{14}{3}+b=0$$

$$\Rightarrow \quad \frac{4a+42+9b}{9}=0$$

$$\Rightarrow \quad 4a+9b=-42 \quad \text{...(1)}$$

and $a(-3)^2+7(-3)+b=0$

$$\Rightarrow \quad 9a-21+b=0$$

$$\Rightarrow \quad 9a+b=21 \quad \text{...(2)}$$

Multiply eq. (2) by 9 and then subtract eq. (1) from eq. (2),

$$(81a+9b)-(4a+9b)=189+42$$

$$\Rightarrow \quad 77a=231$$

$$\Rightarrow \quad a=\frac{231}{77}=3$$

Put $a=3$ in eq. (1), we get

$$4\times3+9b=-42$$

$$\Rightarrow \quad 12+9b=-42$$

$$\Rightarrow \quad 9b=-54$$

$$\Rightarrow \quad b=\frac{-54}{9}$$

$$\Rightarrow \quad b=-6$$

Hence, $a=3$ and $b=-6$

2. Given quadratic equation is

$$9x^2-6px+(p^2-q^2)=0$$

$$\Rightarrow 9x^2-[3(p-q)+3(p+q)]x+(p-q)(p+q)=0$$

$$\Rightarrow 9x^2-3(p-q)x-3(p+q)x+(p-q)(p+q)=0$$

$$\Rightarrow 3x[3x-(p-q)]-(p+q)[3x-(p-q)]=0$$

$$\Rightarrow [3x-(p-q)][3x-(p+q)]=0$$

$$\Rightarrow 3x-(p-q)=0 \quad \text{or} \quad 3x-(p+q)=0$$

$$\Rightarrow x=\frac{(p-q)}{3} \quad \text{or} \quad x=\frac{(p+q)}{3}$$

3. Given equation is $\sqrt{3}x^2+10x+7\sqrt{3}=0$.

TR!CK

Product $\sqrt{3}\times7\sqrt{3}=21$

$\because$ $21=7\times3$

So, middle term, $10=3+7$

$$\Rightarrow \sqrt{3}x^2+(3+7)x+7\sqrt{3}=0$$

(by splitting middle term)

$$\Rightarrow \sqrt{3}x^2+3x+7x+7\sqrt{3}=0$$

$$\Rightarrow \sqrt{3}x(x+\sqrt{3})+7(x+\sqrt{3})=0$$

$$\Rightarrow (\sqrt{3}x+7)(x+\sqrt{3})=0$$

$$\Rightarrow (\sqrt{3}x+7)=0 \quad \text{or} \quad (x+\sqrt{3})=0$$

$$\Rightarrow \quad x=\frac{-7}{\sqrt{3}} \quad \text{or} \quad x=-\sqrt{3}$$

4. Given quadratic equation is

$$x^2+2\sqrt{2}x-6=0$$

On comparing with $ax^2+bx+c=0$, we get

$$a=1, b=2\sqrt{2} \text{ and } c=-6$$

Using quadratic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2\sqrt{2}\pm\sqrt{(2\sqrt{2})^2-4\times1\times(-6)}}{2\times1}$$

$$=\frac{-2\sqrt{2}\pm\sqrt{8+24}}{2}=\frac{-2\sqrt{2}\pm\sqrt{32}}{2}$$

$$=\frac{-2\sqrt{2}\pm4\sqrt{2}}{2}=-\sqrt{2}\pm2\sqrt{2}$$

$$\Rightarrow \quad x=-\sqrt{2}+2\sqrt{2} \text{ and } x=-\sqrt{2}-2\sqrt{2}$$

$$\Rightarrow \quad x=\sqrt{2} \quad \text{and } x=-3\sqrt{2}$$

5. Given quadratic equation is $t^2+3t-10=0$.

$$\Rightarrow \quad t^2+(5-2)t-10=0$$

(by splitting the middle term)

$$\Rightarrow \quad t^2+5t-2t-10=0$$

$$\Rightarrow \quad t(t+5)-2(t+5)=0$$

$$\Rightarrow \quad (t-2)(t+5)=0$$

$$\Rightarrow \quad t=2 \text{ or } -5$$

Thus, roots of the given equation are 2 and –5.

Now, the sum of reciprocal of their roots

$$=\frac{1}{2}-\frac{1}{5}=\frac{5-2}{10}=\frac{3}{10}$$

Hence, the sum of reciprocal of their roots is $\frac{3}{10}$.

6. Given quadratic equation is $x^2-14x+24=0$.

TR!CK

$\because$ $24=2\times12=4\times6=8\times3=\ldots$

Here, we take 12 and 2 as a factors of 24.

So, middle term, $-14=-12-2$.

$$\Rightarrow \quad x^2-(12+2)x+24=0$$

(by splitting the middle term)

$$\Rightarrow \quad x^2-12x-2x+24=0$$

$$\Rightarrow x(x-12)-2(x-12)=0$$

$$\Rightarrow \quad (x-2)(x-12)=0$$

$$\Rightarrow \quad x=2 \text{ or } 12$$

Hence, roots of given equation are 2 and 12.

Now, the sum of square of their roots

$$=(2)^2+(12)^2=4+144=148$$

Hence, the sum of square of their roots is 148.

7. Given quadratic equation is

$$mx(5x-6)=0$$

$$\Rightarrow \quad 5mx^2-6mx=0$$

Here $a = 5m$, $b = -6m$, $c = 0$.

Since, equation has equal roots.

$\therefore \quad D = b^2 - 4ac = 0$

$\Rightarrow \quad (-6m)^2 - 4 \times 5m \times 0 = 0$

$\Rightarrow \quad (6m)^2 = 0$

$\Rightarrow \quad m^2 = 0 \Rightarrow m = 0$

8. Given quadratic equation is

$$(m-1)x^2 + 2(m-1)x + 1 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$a = m-1$, $b = 2(m-1)$ and $c = 1$

For equal roots, $D = 0$

$\therefore \quad b^2 - 4ac = 0$

$\Rightarrow \quad [2(m-1)]^2 - 4 \times (m-1)(1) = 0$

$\Rightarrow \quad 4(m-1)^2 - 4 \times (m-1) = 0$

$\Rightarrow \quad 4(m-1)(m-1-1) = 0$

$\Rightarrow \quad 4(m-1)(m-2) = 0$

$\Rightarrow \quad m-1 = 0$ or $m-2 = 0$

$\Rightarrow \quad m = 1$ or $m = 2$

9. Given equation is $(b-c)x^2 + (c-a)x + (a-b) = 0$.

On comparing with $Ax^2 + Bx + C = 0$, we get

$A = b - c$, $B = c - a$ and $C = a - b$

For equal roots, $D = 0$

$\Rightarrow \quad B^2 - 4AC = 0$

$\Rightarrow \quad (c-a)^2 - 4(b-c)(a-b) = 0$

$\Rightarrow \quad c^2 + a^2 - 2ca - 4ba + 4b^2 + 4ca - 4cb = 0$

$\Rightarrow \quad c^2 + a^2 + 4b^2 + 2ca - 4ba - 4cb = 0$

$\Rightarrow \quad (c + a - 2b)^2 = 0$

TR!CK

Using identity

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow \quad c + a - 2b = 0$

or $\quad c + a = 2b$ **Hence proved.**

10. Let present age of Ritu be x yr. Then her age when she was 5 yr younger $= (x-5)$ yr

According to the given condition,

Square of her age $=$ 11 more than 5 times her present age

$\therefore \quad (x-5)^2 = 5x - 11$

$\Rightarrow \quad x^2 + 25 - 10x = 5x - 11$

$\Rightarrow \quad x^2 - 15x + 36 = 0$

$\Rightarrow \quad x^2 - (12 + 3)x + 36 = 0$

$\Rightarrow \quad x^2 - 12x - 3x + 36 = 0$

$\Rightarrow \quad x(x-12) - 3(x-12) = 0$

$\Rightarrow \quad (x-3)(x-12) = 0$

$\Rightarrow \quad x - 3 = 0$ or $x - 12 = 0$

$\Rightarrow \quad x = 3$ or $x = 12$

At $x = 3$, age is negative, which is not possible.

Hence, Ritu's present age is 12 yr.

11. Let present age of Rehan be x yr.

According to the given condition,

$(x-5)(x+7) = 1 + 2x$

$\Rightarrow \quad x^2 + 2x - 35 = 1 + 2x$

$\Rightarrow \quad x^2 - 36 = 0$

$\Rightarrow \quad x = \pm 6$

$\therefore \quad x = 6$ ($\because$ age is always positive, so neglect negative value)

Hence, present age of Rehan is 6 yr.

12. Given equation is $6x^2 - bx + 2 = 0$.

On comparing with $Ax^2 + Bx + C = 0$, we get

$A = 6$, $B = -b$ and $C = 2$

$\therefore$ Discriminant of given equation is

$D = B^2 - 4AC$

$= (-b)^2 - 4 \times 6 \times 2 = b^2 - 48$

But, it is given that $D = 1$

$\therefore \quad 1 = b^2 - 48$

$\Rightarrow \quad b^2 = 49$

$\Rightarrow \quad b = \pm 7$

Hence, the positive value of b is 7. $\quad (\because b \neq -7)$

Put $b = 7$ in the given equation, then

$6x^2 - 7x + 2 = 0$

$\Rightarrow \quad 6x^2 - (4 + 3)x + 2 = 0$ (by splitting the middle term)

$\Rightarrow \quad 6x^2 - 4x - 3x + 2 = 0$

$\Rightarrow 2x(3x-2) - 1(3x-2) = 0$

$\Rightarrow \quad (2x-1)(3x-2) = 0$

$\Rightarrow (2x-1) = 0$ or $3x - 2 = 0$

$\Rightarrow \quad x = \frac{1}{2}$ or $x = \frac{2}{3}$

Hence, roots of given equation are $\frac{1}{2}$ and $\frac{2}{3}$.

Short Answer Type-II Questions

1. Given quadratic equation is $x^2 - 7x + 10 = 0$.

Since, α and β are the roots of the given equation.

$\therefore$ Sum of roots $= (-1) \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$\Rightarrow \quad \alpha + \beta = (-1)\frac{(-7)}{1} = 7 \quad$ ...(1)

and product of roots $= (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\Rightarrow \quad \alpha \cdot \beta = 1 \cdot \frac{10}{1} = 10 \quad$ ...(2)

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (7)^2 - 2 \times 10$

$= 49 - 20 = 29.$

and $\quad \alpha^2 . \beta^2 = (\alpha\beta)^2 = (10)^2 = 100$

So, the required quadratic equation whose roots are α^2 and β^2, is:

$x^2 - (\alpha^2 + \beta^2)x + (\alpha^2\beta^2) = 0$

$\Rightarrow \quad x^2 - 29x + 100 = 0$

2. Given, $\quad \frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

$\Rightarrow \quad \frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30}$

$$\Rightarrow \quad \frac{x-7-x-4}{x^2-3x-28}=\frac{11}{30}$$
$\Rightarrow \quad (x^2-3x-28)\times 11=-11\times 30$
$\Rightarrow \quad x^2-3x-28=-30$
$\Rightarrow \quad x^2-3x-28+30=0$
$\Rightarrow \quad x^2-3x+2=0$
$\Rightarrow \quad x^2-2x-x+2=0$
$\Rightarrow \quad x(x-2)-1(x-2)=0$
$\Rightarrow \quad (x-2)(x-1)=0$
$\Rightarrow \quad x-2=0$ or $x-1=0$
$\Rightarrow \quad x=2$ or $x=1$

COMMON ERR(!)R

Students do error in simplifying these type of equations. So, adequate practice is required.

3. Given equation is
$(1+m^2)x^2+2mcx+c^2-a^2=0$
On comparing with $Ax^2+Bx+C=0$, we get
$A=(1+m^2)$, $B=2mc$ and $C=c^2-a^2$
$\therefore$ Discriminant $(D)=B^2-4AC$
$=(2mc)^2-4(1+m^2)(c^2-a^2)$
$=4m^2c^2-4(c^2+c^2m^2-a^2-a^2m^2)$
$=4(a^2+a^2m^2-c^2)$
If the given quadratic equation has equal roots,
then $\quad D=0$
$\Rightarrow \quad 4(a^2+a^2m^2-c^2)=0$
$\Rightarrow \quad c^2=a^2(1+m^2)$ **Hence proved.**

4.

TiP

A real number α is said to be a root of an equation $ax^2+bx+c=0$, if $a\alpha^2+b\alpha+c=0$.

Since, 3 is a root of $x^2-x+k=0$.
$\therefore \quad 3^2-3+k=0$
$\Rightarrow \quad 9-3+k=0 \Rightarrow k=-6$
Now, on substituting $k=-6$ in $x^2+k(2x+k+2)+p=0$, we get
$x^2+(-6)\{2x+(-6)+2\}+p=0$
$\Rightarrow \quad x^2-12x+24+p=0$
On comparing with $ax^2+bx+c=0$, we get
$a=1, b=-12, c=24+p$
For equal roots, $D=0 \Rightarrow b^2-4ac=0$
$\therefore \quad (-12)^2-4\times 1\times(24+p)=0$
$\Rightarrow \quad 144-96-4p=0$
$\Rightarrow \quad 48-4p=0 \Rightarrow 4p=48$
$\Rightarrow \quad p=\frac{48}{4}=12$
Hence, the value of p is 12.

5. Let the required consecutive odd numbers be x and $x+2$.
According the to given condition,
$x^2+(x+2)^2=394$
$\Rightarrow \quad x^2+x^2+4x+4=394$
$\Rightarrow \quad 2x^2+4x-390=0$
$\Rightarrow \quad x^2+2x-195=0$
$\Rightarrow \quad x^2+15x-13x-195=0$
$\Rightarrow \quad x(x+15)-13(x+15)=0$
$\Rightarrow \quad (x+15)(x-13)=0$
$\Rightarrow \quad x+15=0$ or $x-13=0$
$\Rightarrow \quad x=-15$ or $x=13$
Hence, required consecutive odd numbers are -15 and -13 or 13 and 15.

6. Given, the sum of the ages of a father and his son is 45 years.
Let age of father and his son be x years and $(45-x)$ years respectively.
Now, five years ago, age of father $=(x-5)$ years
and five years ago, age of his son $=(45-x-5)$
$=(40-x)$ years
According to given condition,
$\Rightarrow \quad (x-5)(40-x)=124$
$\Rightarrow \quad 40x-200-x^2+5x=124$
$\Rightarrow \quad -x^2+45x-200=124$
$\Rightarrow \quad x^2-45x+324=0$
$\Rightarrow \quad x^2-36x-9x+324=0$
(by splitting the middle term)
$\Rightarrow \quad x(x-36)-9(x-36)=0$
$\Rightarrow \quad (x-36)(x-9)=0$
$\Rightarrow \quad x-36=0$ or $x-9=0$
$\Rightarrow \quad x=9, 36$
Here $x=9$ cannot be possible, because at $x=9$, age of his son is $45-9=36$ years, which cannot be possible.
$\therefore \quad x=36 \quad [\because x\neq 9]$
Thus, present age of father $=x$ years $=36$ years and present age of his son $=(45-x)$ years $=(45-36)$
$=9$ years.

7. Let the present age of Roohi be x year.
Three years ago, Roohi's age $=(x-3)$ years
and five years hence, Roohi's age $=(x+5)$ years
According to given condition,
$$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$
$$\Rightarrow \quad \frac{(x+5)+(x-3)}{(x-3)(x+5)}=\frac{1}{3}$$
$\Rightarrow \quad 3(2x+2)=x^2-3x+5x-15$
$\Rightarrow \quad 6x+6=x^2+2x-15$
$x^2-4x-21=0$
$\Rightarrow \quad x^2-7x+3x-21=0$ (by splitting the middle term)
$\Rightarrow \quad x(x-7)+3(x-7)=0$
$\Rightarrow \quad (x-7)(x+3)=0$
$\Rightarrow \quad x-7=0$ or $x+3=0$
$\Rightarrow \quad x=7, -3$
But age cannot be negative.
$\therefore \quad x=7$
Hence, Roohi's present age is 7 years.

8. Let the required natural numbers be x and $15 - x$.

According to the given condition,

$$\frac{1}{x} + \frac{1}{15-x} = \frac{3}{10}$$

$$\Rightarrow \quad \frac{15-x+x}{x(15-x)} = \frac{3}{10} \Rightarrow \frac{15}{(15x-x^2)} = \frac{3}{10}$$

$$\Rightarrow \quad 15x - x^2 = 50 \Rightarrow x^2 - 15x + 50 = 0$$

$$\Rightarrow \quad x^2 - 10x - 5x + 50 = 0$$

(by splitting the middle term)

$$\Rightarrow \quad x(x-10) - 5(x-10) = 0$$

$$\Rightarrow \quad (x-10)(x-5) = 0$$

$$\Rightarrow \quad x - 10 = 0 \quad \text{or} \quad x - 5 = 0$$

$$\Rightarrow \quad x = 10 \quad \text{or} \quad x = 5$$

When $x = 10$ then other number is $(15 - 10) = 5$.

When $x = 5$ then other number is $(15 - 5) = 10$.

Hence, required numbers are (10, 5) or (5, 10).

9. Let the required natural numbers be x and $x + 3$.

According to the given condition,

$$\frac{1}{x} - \frac{1}{x+3} = \frac{3}{28} \Rightarrow \frac{x+3-x}{x(x+3)} = \frac{3}{28}$$

$$\Rightarrow \quad 3 \times 28 = 3 \times x(x+3)$$

$$\Rightarrow \quad 28 = x^2 + 3x$$

$$\Rightarrow \quad x^2 + 3x - 28 = 0$$

$$\Rightarrow \quad x^2 + 7x - 4x - 28 = 0$$

$$\Rightarrow \quad x(x+7) - 4(x+7) = 0$$

$$\Rightarrow \quad (x+7)(x-4) = 0$$

$$\Rightarrow \quad x + 7 = 0 \text{ or } x - 4 = 0$$

$$\Rightarrow \quad x = -7 \text{ or } x = 4$$

But $x \neq -7$ (rejecting) because -7 is not a natural number.

$$\therefore \quad x = 4$$

and $$x + 3 = 4 + 3 = 7$$

Hence, required numbers are 4 and 7.

10. Let the original speed of the aeroplane be x km/h.

So, time taken to cover 1500 km distance

$$(T_1) = \frac{1500}{x}$$

New speed of aeroplane = $(x + 100)$ km/h

So, new time $(T_2) = \dfrac{1500}{x+100}$

According to the given condition, $T_1 - T_2 = 30$ min

$$\Rightarrow \quad \frac{1500}{x} - \frac{1500}{x+100} = \frac{30}{60} = \frac{1}{2}\text{h}$$

$$\Rightarrow \quad 1500\left[\frac{1}{x} - \frac{1}{x+100}\right] = \frac{1}{2}$$

$$\Rightarrow \quad 1500\left[\frac{x+100-x}{x(x+100)}\right] = \frac{1}{2}$$

$$\Rightarrow \quad x(x+100) = 2 \times 1500 \times 100$$

$$\Rightarrow \quad x^2 + 100x - 300000 = 0$$

$$\Rightarrow \quad x^2 + 600x - 500x - 300000 = 0$$

$$\Rightarrow \quad x(x+600) - 500(x+600) = 0$$

$$\Rightarrow \quad (x+600)(x-500) = 0$$

$$\Rightarrow \quad x + 600 = 0 \text{ or } x - 500 = 0$$

$$\Rightarrow \quad x = -600 \text{ (rejecting)} \quad \text{or} \quad x = 500$$

Since, the speed of aeroplane cannot be negative.

Hence, the original speed of the plane is 500 km/h.

COMMON ERR(!)R

Students should aware of taking the value of time. Sometimes students take the value of time 30 min as it is. While this time should be converted into hour.

Long Answer Type Questions

1. Given equation is $\dfrac{1}{x+1} + \dfrac{2}{x+2} = \dfrac{4}{x+4}$.

TiP

Adequate practice is necessary for simplifying these type of equations.

$$\Rightarrow \quad \frac{x+2+2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4}$$

$$\Rightarrow \quad \frac{x+2+2x+2}{x^2+3x+2} = \frac{4}{x+4}$$

$$\Rightarrow \quad (x+4)(3x+4) = 4(x^2+3x+2)$$

$$\Rightarrow \quad 3x^2 + 4x + 12x + 16 = 4x^2 + 12x + 8$$

$$\Rightarrow \quad 4x^2 - 3x^2 + 12x - 16x + 8 - 16 = 0$$

$$\Rightarrow \quad x^2 - 4x - 8 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, \; b = -4 \; \text{and} \; c = -8$$

Using quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

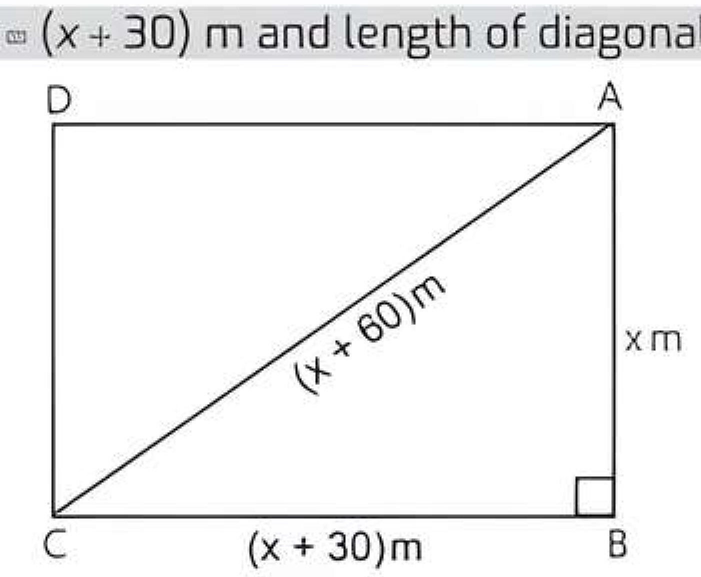
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-8)}}{2 \times 1}$$

$$= \frac{4 \pm \sqrt{16+32}}{2} = \frac{4 \pm \sqrt{48}}{2}$$

$$= \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$$

Hence, the required roots are $2 + 2\sqrt{3}$ and $2 - 2\sqrt{3}$.

2. Let the shorter side of the field be x m. Then, longer side = $(x + 30)$ m and length of diagonal = $(x + 60)$ m



In right angled Δ ABC, by Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (x+60)^2 = x^2 + (x+30)^2$$

$$\Rightarrow \quad x^2 + 3600 + 120x = x^2 + x^2 + 900 + 60x$$

$$\Rightarrow \quad x^2 + 60x - 120x + 900 - 3600 = 0$$

$$\Rightarrow \quad x^2 - 60x - 2700 = 0$$

$$\Rightarrow \quad x^2 - 90x + 30x - 2700 = 0$$

(by splitting the middle term)

$\Rightarrow \quad x(x-90)+30(x-90)=0$

$\Rightarrow \quad (x-90)(x+30)=0$

$\Rightarrow \quad x-90=0 \text{ or } x+30=0$

$\Rightarrow \quad x=90$

or $x=-30$ (rejecting, because length of the side cannot be negative)

Hence, length of shorter side is 90 m and length of longer side is $90+30=120$ m.

COMMON ERROR

Some students do not know how to frame the equation.

3. Let the smaller number be x and larger number be y. Then,

$$y^2-x^2=180 \quad ...(1)$$

and

$$x^2=8y \quad ...(2)$$

$\therefore \quad y^2-8y=180$

$\Rightarrow \quad y^2-8y-180=0$

$\Rightarrow \quad y^2-18y+10y-180=0$

$\Rightarrow \quad y(y-18)+10(y-18)=0$

$\Rightarrow \quad (y+10)(y-18)=0$

$\Rightarrow \quad y+10=0 \text{ or } y-18=0$

$\Rightarrow \quad y=-10 \text{ or } y=18$

$\therefore \quad y=18$

($\because$ negative value of y cannot satisfy it)

Put $y=18$ in eq. (2),

$x^2=8\times18$

$\Rightarrow \quad x^2=144 \Rightarrow x=12$

Hence, two numbers are 12 and 18.

4. Let the side of first square $=x$ m.

Then its perimeter $=4x$ m

$\because$ The sum of the perimeter of two squares is 68 m.

$\therefore$ Perimeter of second square $=(68-4x)$ m

Then side of second square

$$=\left(\frac{68-4x}{4}\right)\text{m}=(17-x)\text{m}$$

$\therefore$ Area of first square $=x^2$ m^2

and area of second square $=(17-x)^2$ m^2

$=(289+x^2-34x)$ m^2

According to the question,

Sum of the areas of two squares $=157$ m^2

$\therefore \quad x^2+289+x^2-34x=157$

$\Rightarrow \quad 2x^2-34x+132=0$

$\Rightarrow \quad x^2-17x+66=0$

$\Rightarrow \quad x^2-11x-6x+66=0$

$\Rightarrow \quad x(x-11)-6(x-11)=0$

$\Rightarrow \quad (x-11)(x-6)=0$

$\Rightarrow \quad x=11 \text{ or } x=6$

If $x=11$, then the side of first square $=11$ m

and the side of second square $=17-11=6$ m

If $x=6$, then the side of first square $=6$ m

and the side of second square $=17-6=11$ m

Hence, the sides of the squares are 6 m and 11 m.

COMMON ERROR

Some students do not know how to frame the equation and some frame it correctly but fail to solve it.

5. Let three consecutive integers be x, $(x+1)$ and $(x+2)$.

According to the given condition,

Square of first number + Product of second and third consecutive number = 154

$\therefore \quad x^2+(x+1)(x+2)=154$

$\Rightarrow \quad x^2+x^2+3x+2=154$

$\Rightarrow \quad 2x^2+3x-152=0$

On comparing with $ax^2+bx+c=0$, we get

$a=2, b=3$ and $c=-152$

By using quadratic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-3\pm\sqrt{(3)^2-4\times2\times(-152)}}{2\times2}$$

$$=\frac{-3\pm\sqrt{9+1216}}{4}=\frac{-3\pm\sqrt{1225}}{4}=\frac{-3\pm35}{4}$$

$\therefore \quad x=\frac{-3+35}{4} \text{ or } x=\frac{-3-35}{4}$

$\Rightarrow \quad x=\frac{32}{4} \text{ or } x=\frac{-38}{4}$

$\Rightarrow \quad x=8 \text{ or } x=\frac{-38}{4}$

('–ve' neglecting, because it is not positive integer)

So, we consider only $x=8$.

Therefore, next two consecutive integers are

$x+1=8+1=9$

and $x+2=8+2=10$

Hence, three consecutive integers are 8, 9 and 10.

6. Let speed of stream be x km/h.

$\therefore$ Speed of boat in upstream $=(18-x)$ km/h

and speed of boat in downstream $=(18+x)$ km/h

Now, time taken to go upstream

$$=\frac{\text{distance}}{\text{speed}}=\frac{24}{18-x}\text{ h}$$

Similarly, time taken to go downstream $=\frac{24}{18+x}$ h

According to the question,

$$\frac{24}{18-x}-\frac{24}{18+x}=1$$

$\Rightarrow \quad \frac{18+x-18+x}{324-x^2}=\frac{1}{24}$

$\Rightarrow \quad 24\times2x=324-x^2$

$\Rightarrow \quad x^2+48x-324=0$

$\Rightarrow \quad x^2+54x-6x-324=0$

$\Rightarrow \quad x(x+54)-6(x+54)=0$

$\Rightarrow \quad (x+54)(x-6)=0$

$\Rightarrow \quad x=-54 \text{ or } x=6$

Since, x is the speed of stream, so it cannot be negative.

$\therefore \quad x=6 \quad (\because x\neq-54)$

Hence, speed of stream is 6 km/h.

COMMON ERR(!)R

Students don't be confused by taken the speed of stream in upstream and downstream. Sometimes students take the speed of stream in upstream and downstream as $(r + x)$ km/h and $(r - x)$ km/h respectively, which is wrong. The correct speed of stream in upstream and downstream are $(r - x)$ km/h and $(r + x)$ km/h respectively.

7. Let the speed of the train be x km/h.

TR!CK

$$Time = \frac{distance}{speed}$$

Time required to travel the distance 360 km

$$= \frac{360}{x} \text{h}.$$

If the speed of the train is 5 km/h more *i.e.*, $(x + 5)$ km/h, then

Time required to travel the distance 360 km

$$= \frac{360}{x+5} \text{h}$$

$\because$ This time is 1 h less than the before time.

$$\therefore \quad \frac{360}{x} - \frac{360}{x+5} = 1$$

$$\Rightarrow \quad \frac{360x + 1800 - 360x}{x(x+5)} = 1$$

$$\Rightarrow \quad \frac{1800}{x^2 + 5x} = 1 \Rightarrow x^2 + 5x = 1800$$

$$\Rightarrow \quad x^2 + 5x - 1800 = 0$$

$$\Rightarrow x^2 + 45x - 40x - 1800 = 0$$

[by splitting the middle term]

$$\Rightarrow \quad x(x + 45) - 40(x + 45) = 0$$

$$\Rightarrow \quad (x + 45)(x - 40) = 0$$

$$\Rightarrow \quad x + 45 = 0 \text{ or } x - 40 = 0$$

$$\Rightarrow \quad x = -45, 40$$

But speed of train cannot be negative.

$\therefore$ The value of x *i.e.*, -45 is not acceptable.

So, $x = 40$

Hence, the speed of train is 40 km/h.

8. Let the time taken by the smaller pipe to fill the tank be x hour.

Time taken by the larger pipe $= (x - 10)$ hour

Part of tank filled by smaller pipe in 1 hour

$$= \frac{1}{x}$$

Part of tank filled by larger pipe in 1 hour

$$= \frac{1}{x - 10}$$

It is given that the tank can filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together.

Therefore,

$$\frac{1}{x} + \frac{1}{x - 10} = \frac{8}{75}$$

$$\Rightarrow \quad \frac{x - 10 + x}{x(x - 10)} = \frac{8}{75} \Rightarrow \frac{2x - 10}{x(x - 10)} = \frac{8}{75}$$

$$\Rightarrow \quad 75(2x - 10) = 8x^2 - 80x$$

$$\Rightarrow \quad 150x - 750 = 8x^2 - 80x$$

$$\Rightarrow \quad 8x^2 - 230x + 750 = 0$$

$$\Rightarrow \quad 8x^2 - 200x - 30x + 750 = 0$$

$$\Rightarrow \quad 8x(x - 25) - 30(x - 25) = 0$$

$$\Rightarrow \quad (x - 25)(8x - 30) = 0$$

$$\Rightarrow \quad x = 25, \frac{30}{8}$$

Time taken by the smaller pipe cannot be $\frac{30}{8} = 3.75$ hours. As in this case, the time taken by the larger pipe will be negative, which is not possible.

Hence, time taken individually by the smaller pipe and the larger pipe will be 25 and $25 - 10 = 15$ hours respectively.

COMMON ERR(!)R

Some students do not know how to frame the equation. Some frame it correctly but fail to solve it.

Chapter Test

Multiple Choice Questions

Q 1. If $x^2 + k(4x + k - 1) + 2 = 0$ has equal roots, then values of k are:

a. $\frac{2}{3}, -1$ b. $-\frac{2}{3}, 1$ c. $-\frac{2}{3}, -1$ d. $\frac{1}{3}, \frac{1}{4}$

Q 2. The roots of the quadratic equation $2x^2 - 7x + 3 = 0$ are:

a. $3, -\frac{1}{2}$ b. $3, \frac{1}{2}$ c. $\frac{1}{2}, \frac{1}{3}$ d. $-2, -3$

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): The value of $k = -\frac{1}{4}$, if one root of the quadratic equation $7x^2 - x + 5k = 0$ is $\frac{1}{2}$.

Reason (R): The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has atmost two roots.

Q 4. Assertion (A): The equation $x^2 + kx + 4 = 0$ has equal roots for $k = \pm 4$.

Reason (R): If discriminant 'D' of a quadratic equation equals to zero, then the roots of quadratic equation are real and equal.

Fill in the Blanks

Q 5. If $\frac{1}{4}$ is a root of the equation $3x^2 + kx - \frac{5}{2} = 0$, then the value of k is ....

Q 6. The nature of roots of quadratic equation $5x^2 + 3x - 7 = 0$ has .... and ....

True/False

Q 7. The roots of the equation $x^2 + kx - 16 = 0$ has equal and opposite sign when $k = 0$.

Q 8. If any quadratic equation has discriminant $D > 0$, then roots are real and distinct.

Case Study Based Question

Q 9. Raj and Ajay are very close friends. Both of them decide to go to Ranikhet by their own cars. Raj's car runs at a speed of x km/h while Ajay's car runs 5 km/h faster than Raj's car. Raj took 4 h more than Ajay to complete the journey of 400 km.

Based on the above information, solve the following questions:

(i) Form the quadratic equation to describe the speed of Raj's car.

(ii) What is the speed of Raj's car?

(iii) How much time took Raj to travel 400 km?

OR

How much time took Ajay to travel 400 km?

Very Short Answer Type Questions

Q 10. Solve for x: $5x + \frac{1}{x} = 0,\ x \neq 0$.

Q 11. Find the value of k for which the equation $4x^2 + kx + 9 = 0$ has real and equal roots.

Short Answer Type-I Questions

Q 12. One year ago, a man was 8 times as old as his son. Now, his age is equal to the square of his son's age. Find their present ages.

Q 13. Find the roots of the equation $ax^2 + a = a^2x + x$.

Short Answer Type-II Questions

Q 14. If the equation $(1 + m^2)\,x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, then show that $c^2 = a^2\,(1 + m^2)$.

Q 15. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

Long Answer Type Question

Q 16. In a rectangular park of dimensions 50 m × 40 m, a rectangular pond is constructed so that the area of grass strip of uniform width surrounding the pond would be 1184 m^2. Find the length and breadth of the pond.